

BRIDGE HOUSE

MATHEMATICS DEPARTMENT

Grade 12: Advanced Programme Mathematics

September Examination 2014

Time: 180 min 300 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 13 pages. Section A consists of 10 questions, Section B consists of 5 questions and Section C consists of 7 questions.

2. Section A is compulsory. Choose either Section B or Section C to complete.

3. An answer sheet is included. Please check that your paper is complete.

4. A formula sheet that will be handed out separately consists of 4 pages.

5. Read the questions carefully.

6. Answer all the questions.

7. Number your answers exactly as the questions are numbered.

8. Round off your answers to **two decimal digits** where necessary.

9. All the necessary working details must be clearly shown.

10. Approved non-programmable calculators may be used unless otherwise stated.

11. It is in your own best interest to present your work neatly.

**Section A Algebra and Calculus (200 marks)**

**Question 1**

Prove by Mathematical Induction that:

for all . **[12]**

**Question 2**

2.1 Solve for :

 2.1.1 (4)

 2.1.2 (4)

 2.1.3 (4)

 2.1.4 (4)

 2.1.5 (5)

2.2 Given , determine in terms of and the value of:

 (7)

 **[28]**

**Question 3**

3.1 If is a zero of , find the other solutions of (8)

3.2 Calculate the value of and if:

 (7)

 **[15]**

**Question 4**

 is continuous for all .

4.1 Calculate possible values for and . (8)

4.2 Is differentiable at ? Use calculations to explain your answer. (4)

 **[12]**

**Question 5**

Draw a rough sketch of the following on the same set of axes:

5.1 (6)

5.2 (9)

 **[15]**

**Question 6**

6.1 Express the following as partial fractions:

 (10)

6.2 Draw a neat sketch of the following curve on the axes provided in the answer sheet, showing all asymptotes and intercepts with the axes.

 (15)

 **[25]**

**Question 7**

Two semi-circles are drawn. is the diameter of the big semi-circle and is the diameter of the small semi-circle.

 is the midpoint.

The radius of the small semi-circle is times the radius of the big semi-circle.

7.1 If the arc length units, Calculate length . (6)

7.2 Calculate the area of the shaded region . (7)

 **[13]**

**Question 8**

A rectangular box has length of , breadth of and height .

8.1 If the surface area of the box is , show that the volume can be given as: . (10)

8.2 Calculate the dimensions of the box that would result in a maximum volume. (5)

 **[15]**

**Question 9**

9.1 Given

9.1.1 Calculate (6)

 9.1.2 Find the tangent to the curve at the point . (6)

9.2 Find the area of the region bounded by and for .

 The figure is given below:



(13)

 **[25]**

**Question 10**

10.1

Find the volume generated by rotating about the -axis. (8)

10.2 Integrate the following:

 10.2.1 (7)

 10.2.2 (9)

10.3 Integrate the following using the given substitution: (Show all working)

 (16)

 **[40]**

**Total for Section A: [200]**

**Section B Finance and Modelling (100 marks)**

**Question 1**

A swimming pool is being treated for an algae build-up using Product A. Initially of the pool is covered in algae. With each day of the treatment, algae reduces by .

1.1 In theory, will the algae ever disappear? Explain (2)

1.2 Calculate the area covered in algae after 7 days. (4)

1.3 After 7 days a new product, Product B, is being tested to treat the algae problem. The rate of algae reduction changes such that in the following days, half the remaining algae is left. What is the new rate at which the algae reduces? (5)

1.4 Which product works faster to reduce algae? (1)

  **[12]**

**Question 2**

 years ago Jack bought a house worth . Before he bought the house, he looked at financing options available:

Option A

Take out a loan over a fixed period of years. Under this loan agreement, repayments will increase if the interest rate increases. First payment is made at the beginning of the first month and last payment is made at the end of the last month.

Option B

Take out a fixed loan with a fixed monthly repayment of per month. Change in the interest rate will have an influence on the number of payments. First payment will occur at the end of the first month.

When Jack bought the house he did not know what would happen to the interest rate, so he decided to go with Option B.

The initial interest rate was compounded monthly. years after that, the interest rate increased to compounded monthly. The interest rate did not change again for the next years.

2.1 Calculate the following from Option A.

 2.1.1 His initial monthly repayment. (5)

 2.1.2 His balance outstanding after years. (6)

 2.1.3 His monthly repayment for the next years. (8)

2.2 Calculate the following for Option B. Assume the balance outstanding after years is .

 2.2.1 His actual remaining period from the end of years to pay off the loan. (8)

 2.2.2 His final payment (It will be less than his fixed payment of ). (7)

2.3 Knowing what Jack knows now, do you think it was a wise decision to go with Option B? Explain using calculations. (4)

 **[38]**

**Question 3**

If and , calculate the value of .

**[8]**

**Question 4**

In a secluded forest, the rabbit ( is the main prey of the fox (). A predator-prey model has been used to model the interaction between these two species in the forest. The period between interactions is year. The equations used to model the populations are:

4.1 What is the average life span of a fox? (3)

4.2 What is the per capita rate of deadly attacks on the rabbit population? (1)

4.3 Calculate the efficiency for foxes to turn prey into cubs? (2)

The graph below shows the population of the Foxes and Rabbits.

Foxes

Rabbits

4.4 Use the graph to estimate the initial population of both species. (2)

4.5 Use the graph to estimate the stable point (equilibrium point) of the two populations. (2)

4.6 Use the formulas to calculate the actual stable point (equilibrium point) of the two populations. (9)

 **[19]**

**Question 5**

The new 3D television has been released. The table below shows the revenue earned by the company over the first 7 months of sales.

|  |  |  |  |
| --- | --- | --- | --- |
| **Months** | **Revenue****(millions)** |  |  |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |
| 6 |  |  |  |
| 7 |  |  |  |

5.1 Complete the table on your answer sheet. (4)

5.2 Plot the values of and on a scatterplot. (5)

5.3 Determine the equation of the line-of-best-fit for the scatterplot. (5)

5.4 Calculate the values of and and write down the logistic equation that would suit this model. (6)

5.5 Compare the predicted revenue of the logistic equation to the actual revenue given in the table. How effective is the equation as a model of the sales of the 3D television? (3)

  **[23]**

**Total for Section B: [100]**

**TOTAL FOR THIS PAPER: [300]**

**Section C Statistics and Probability (100 marks)**

**Question 1**

The final of an ATP Masters Series Event will be contested by Roger Federer (RF) and Novak Djokovic (ND). The match is a best of three set match i.e. the player to win two sets (in any order) wins the match. The probability that the first set is won by RF is . Given that RF wins the first set, the probability that he will win the second set is 0,7. Given that ND wins the first set, the probability that he wins the second is 0,85. Should the match go to a third set, both of them have an equal chance of winning the set, irrespective of who won the second.

1.1 Calculate the probability that ND wins the match. (4)

1.2 Given that ND wins the match, calculate the probability that RF won the first set?

(7)

  **[11]**

**Question 2**

Fifty students are going on a Science field trip. They are to be arranged in two fifteen seater busses and one twenty seater bus.

2.1 In how many different ways can the students be arranged into the three busses? (4)

2.2 Two of the students were involved in a scuffle while they were still in the classroom. The teacher hopes they do not end up riding on the same bus. If the loading of the busses takes place randomly, calculate the probability that these two students will not be on the same bus? (8)

**[12]**

**Question 3**

3.1 A discrete random variable has a pdf of the form for and zero otherwise.

3.1.1 Find the constant value . (6)

3.1.2 Find . (4)

 **[10]**

3.2 A continuous random variable has density function for , and zero otherwise.

3.2.1 Find (7)

3.2.2 Find the median of . (7)

 **[14]**

**Question 4**

Assume the amount of light (in lumens) produced by a certain type of light bulb is normally distributed with mean and variance .

4.1 Find (8)

4.2 Find the value such that the amount of light produced by of the light bulbs will exceed lumens. (8)

 **[16]**

**Question 5**

Mullet (1977) considers the goals scored per game by a few teams in the National Hockey League. The average number of goals scored per game at home and away by each team in the 1973-1974 season was as follows:

|  |  |  |
| --- | --- | --- |
|  | At Home  | Away  |
| Boston | 4,95 | 4,00 |
| Montreal | 4,10 | 3,41 |
| Detroit | 3,64 | 2,56 |
| Vancouver | 4,36 | 2,18 |
| Philadelphia | 3,90 | 3,10 |
| Chicago | 3,64 | 3,33 |
| Los Angeles | 3,36 | 2,62 |
| Atlanta | 3,10 | 2,38 |

5.1 Calculate and interpret the correlation coefficient (r) in ‘real life’ terms. (4)

5.2 Calculate the least squares regression line. (4)

5.3 Predict the average number of goals per away game scored by a team that scored four goals per home game. (3)

**[11]**

**Question 6**

6.1 Consider a random sample of 200 travel insurance policies. Claims were made on 29 of these policies over their most recent year of cover.

Calculate a 99% confidence interval for the proportion of policyholders who make

claims in a given year of cover. (7)

6.2 A large mail-order company has placed an order with its suppliers. If the company wants to estimate the proportion of defective can-openers in the current shipment, correct to within ; with confidence, approximately how large a sample would be required if past records indicate that the proportion of defective can-openers is usually . (9)

 **[16]**

**Question 7**

Gas that is produced from biological fermentation is sold with the claim that the methane

content is normally distributed with a mean of 70% and a standard deviation of 1%. A

random sample of 8 gas canisters gave the following methane contents expressed as a

percentage:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Sample | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| % methane | 64 | 65 | 75 | 67 | 65 | 74 | 75 | 69 |

Conduct a hypothesis test at the 5% significance level to determine whether, based

on this sample, it is fair to claim that the mean methane content is 70%. **[10]**

**Total for Section C: [100]**

**TOTAL FOR THIS PAPER: [300]**

ANSWER SHEET

Section A

**Question 5**

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**Question 6.2**

Section B



|  |  |  |  |
| --- | --- | --- | --- |
| **Months** | **Revenue****(millions)** |  |  |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |
| 6 |  |  |  |
| 7 |  |  |  |

**Question 5.1**

**Question 5.2**

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